About unimodularity of barbells graphs

Cristian Panelo
Universidad Nacional de San Luis, Argentina
crpanelo@unsl.edu.ar

Graphs with a unique perfect matching have been extensively studied in the literature, see [1] and [2]. A graph G is unimodular if $|\det(G)| = 1$. In [3], the problem of characterizing unimodular graphs is proposed, and unicyclic unimodular graphs are characterized. A Kőnig-Egerváry graph is a graph such that its vertex covering number equals its matching number. Kőnig-Egerváry graphs were independently introduced in 1979 by Deming [4] and Sterboul [5]. An even subdivision of a graph G is either the graph G itself or any of the graphs that arise from G by successive application of even subdivisions. A barbell is the graph formed by two disjoint K_3 linked by an edge. We also refer as a barbell graph to any even subdivision of it. In [6], the notion of a barbell part, $\mathrm{B}(G)$, of a graph G with a unique perfect matching was introduced. It was shown that every such graph G can be decomposed into two disjoint subgraphs: $\mathrm{KE}(G)$ (a Kőnig-Egerváry graph) and $\mathrm{B}(G)$ (the subgraph induced by all vertices in M-barbells of G). A graph G is called a B-graph if $\mathrm{B}(G) = G$.

In [6], it was proved that for all graphs with a unique perfect matching:

$$\det(G) = \det(\mathrm{B}(G)) \cdot \det(\mathrm{KE}(G)).$$

Hence, in order to characterize when a graph is unimodular, it is necessary to characterize when Kőnig-Egerváry graphs and B-graphs are unimodular. This work characterizes a large unimodular subfamily of B-graphs.

Trabajo en conjunto con Daniel A Jaume (Universidad Nacional de San Luis) y Diego G Martinez (Universidad Nacional de San Luis).

Referencias

[1] R Simion and D S Cao. Solution to a problem of cd godsil regarding bipartite graphs with unique perfect matching. Combinatorica, 9:85–89, 1989

[2] S Panda and S Pati. On the inverse of a class of bipartite graphs with unique perfect matchings. The Electronic Journal of Linear Algebra, 29:89–101, 2015.

[3] S Akbari and S J Kirkland. On unimodular graphs. Linear Algebra and its Applications, 421(1):3–15, 2007

[4] R W Deming. Independence numbers of graphs - an extension of the Kőnig-Egerváry Theorem. Discrete Mathematics, 27(1):23–33, 1979.

[5] F Sterboul. A characterization of the graphs in which the transversal number equals the matching number. 1979.

[6] D A Jaume, C Panelo, and D G Martinez, Determinantal decomposition of graphs with unique perfect matching, submitted