Maximal Curves and Chebyshev Polynomials over Finite Fields

Saeed Tafazolian

Abstract. Maximal curves over finite fields are curves whose number of rational points attains the Hasse–Weil upper bound, and they often exhibit remarkable arithmetic and geometric structure. In this talk, I will discuss a family of hyperelliptic curves

$$H_d : \qquad y^2 = \varphi_d(x),$$

where φ_d is the reduction of the d-th Chebyshev polynomial over a finite field of odd characteristic. The main question is to determine for which pairs (d, q) the curve H_d is maximal over $\mathbb{F}_{q^2}$.

I will present recent results for prime degree and for general odd d. The methods combine permutation properties of Chebyshev polynomials with the arithmetic of Jacobians, Frobenius and Newton polygons, complex multiplication, and, in a key case, a 2-descent argument. These techniques yield new maximality results, a complete criterion in characteristics congruent to 3 modulo 4 for odd d, and strong restrictions in characteristics congruent to 1 modulo 4. I will conclude with computational evidence and open questions pointing toward a broader classification of maximal curves arising from Chebyshev polynomials.